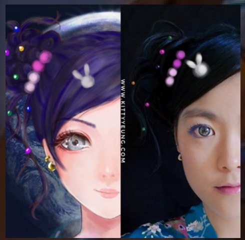
Introduction to Quantum Computing



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April 19, 2020
Hackaday, session 4

Class structure

- [Comics on Hackaday – Introduction to Quantum Computing] every Wed & Sun

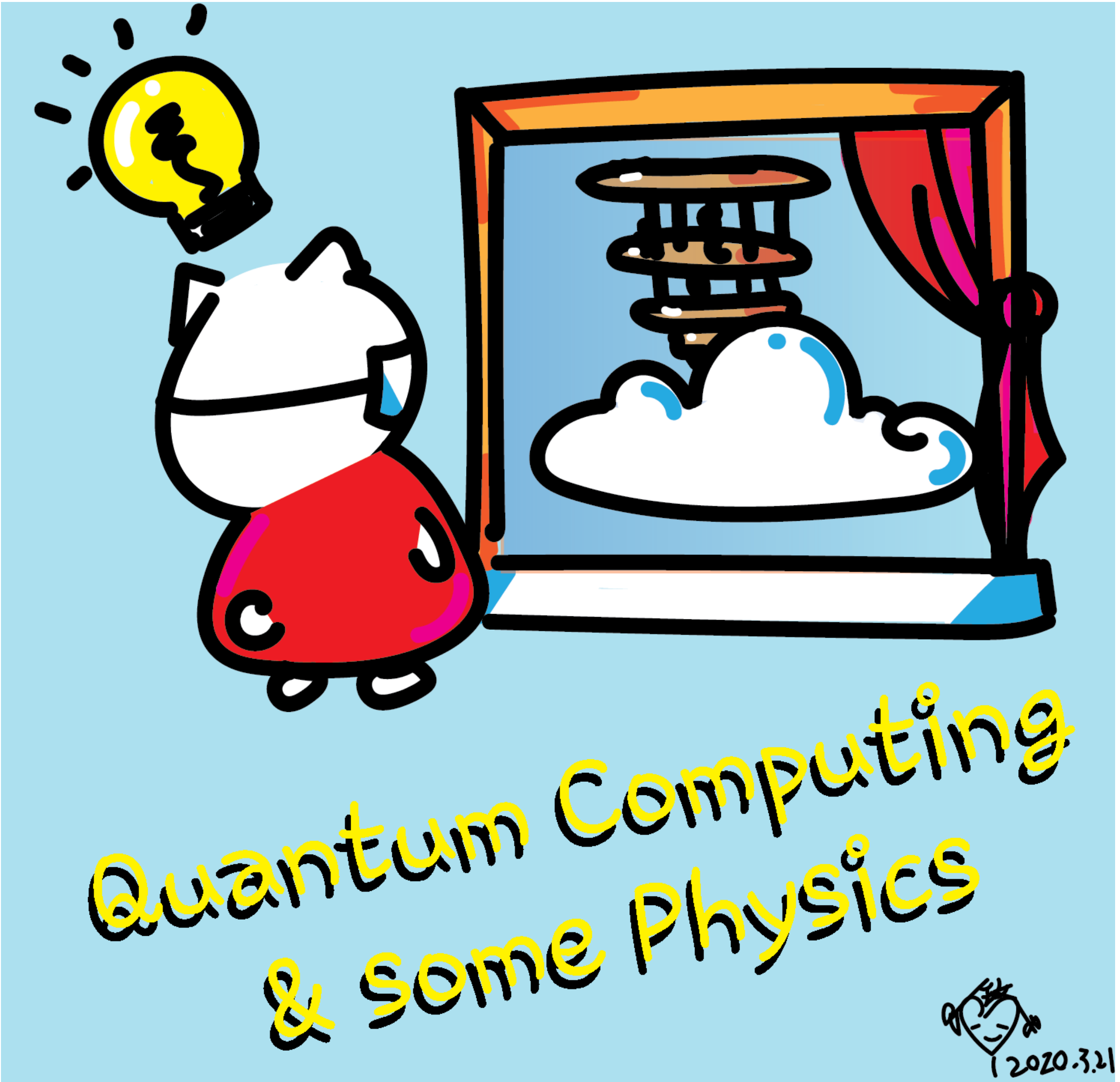
- 30 mins every Sun, one concept (theory, hardware, programming), Q&A

- Contribute to Q# documentation http://docs.microsoft.com/quantum

- Coding through Quantum Katas https://github.com/Microsoft/QuantumKatas/

- Discuss in Hackaday project comments throughout the week

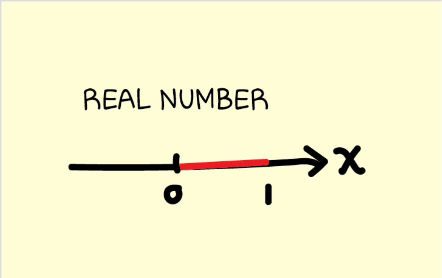
- Take notes



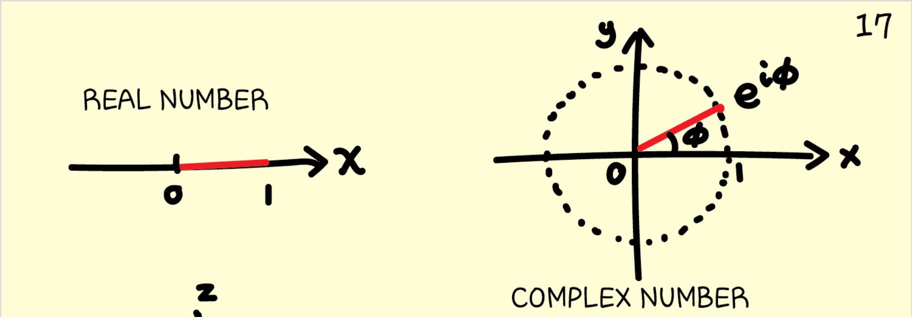
Past concepts

- Superposition
- Interference (measurement result is a result of interference)
- Entanglement (results of entangled qubits are correlated)

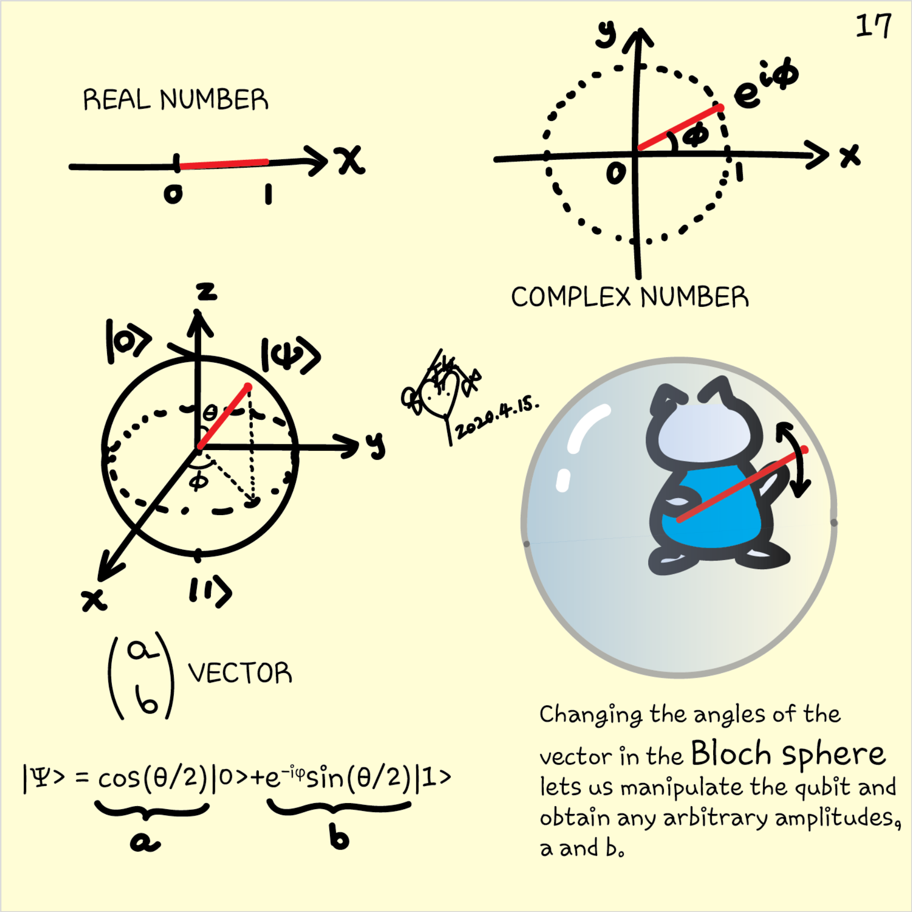
Graphic representation of a qubit



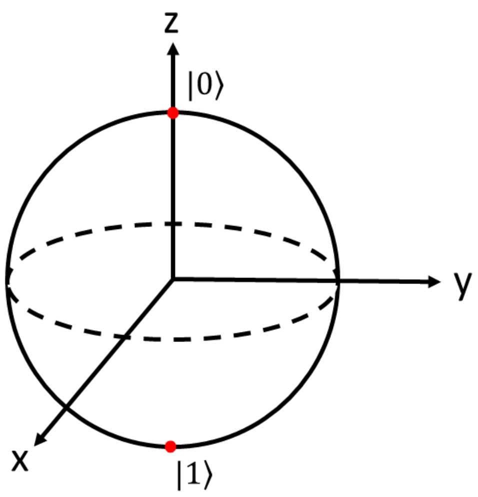
Graphic representation of a qubit

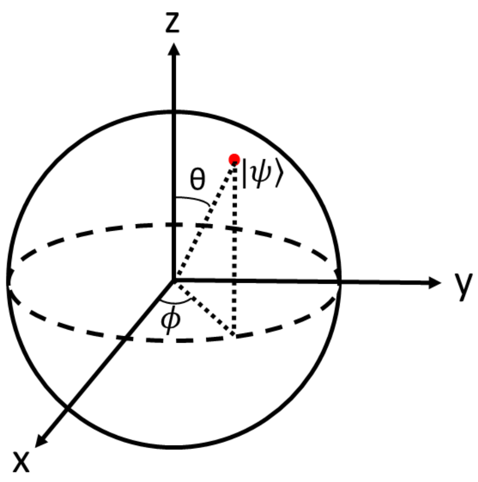


Graphic representation of a qubit



Bloch sphere



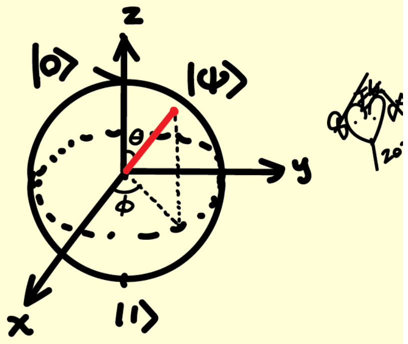


Arbitrary state

$$|\psi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{-i\phi}\sin\frac{\theta}{2}|1\rangle$$

the states $|0\rangle$ and $|1\rangle$ are just two special cases with θ = 0° and 180°, respectively.

Gates (quantum operations)



$\begin{pmatrix} a \\ b \end{pmatrix}$ VECTOR

$$|\Psi\rangle = \underbrace{\cos(\theta/2)}_{a}|0\rangle + \underbrace{e^{-i\varphi}\sin(\theta/2)}_{b}|1\rangle$$

MATRIX THAT CHANGES φ

MATRIX THAT CHANGES θ

18

$$\begin{pmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{-i\phi}\sin\frac{\theta}{2} \end{pmatrix}$$

MATRICES: GATES

VECTOR: QUBIT

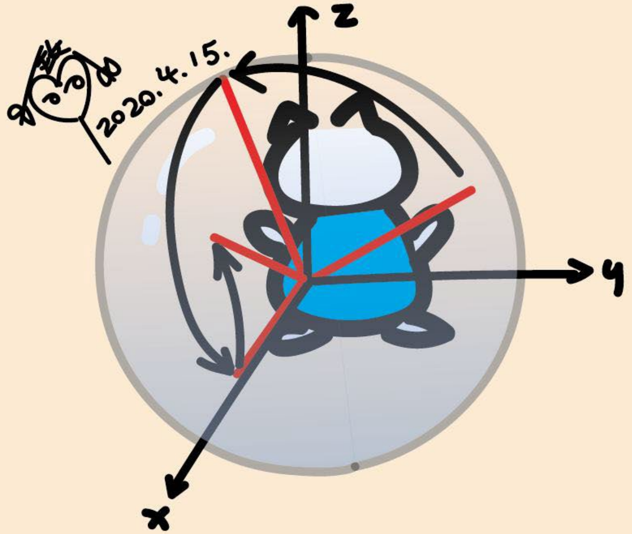
MATRIX THAT CHANGES φ

MATRIX THAT CHANGES θ

$$\begin{pmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} \cos\frac{\theta}{2} \\ e^{-i\phi}\sin\frac{\theta}{2} \end{pmatrix}$$

MATRICES: GATES

VECTOR: QUBIT



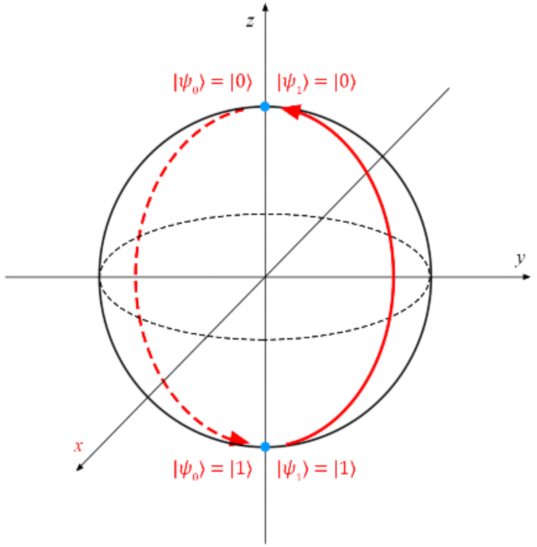
Like a set of coins, a combination of them can make up any number.

We can have a set of matrix operations (gates) that moves the qubit to anywhere on the Bloch sphere.

Pauli gates

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$X \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix}$$



Pauli X

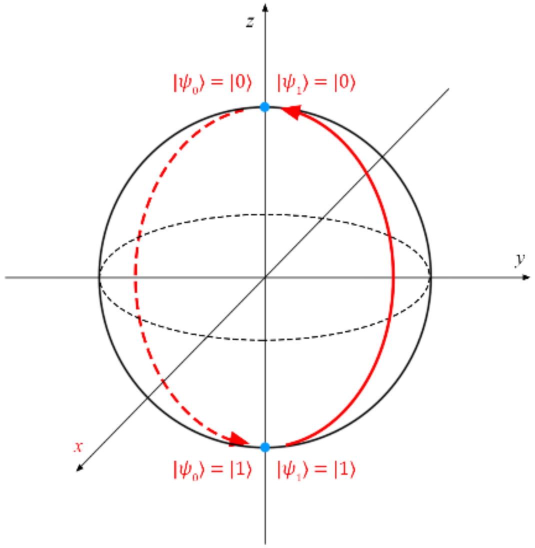
Pauli gates

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

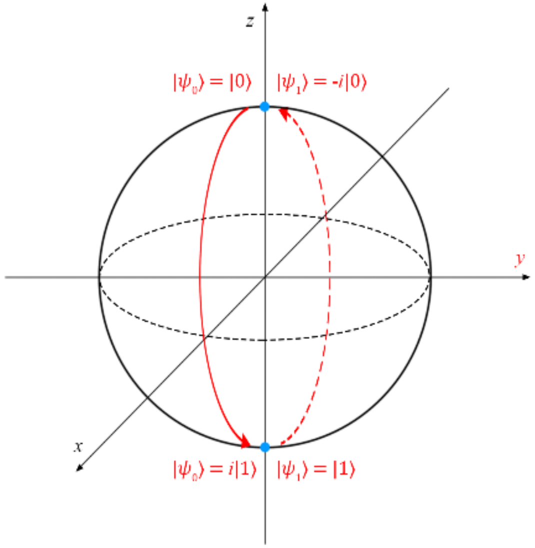
$$Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$X \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix}$$

$$Y \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = i \begin{pmatrix} -\beta \\ \alpha \end{pmatrix}$$



Pauli X

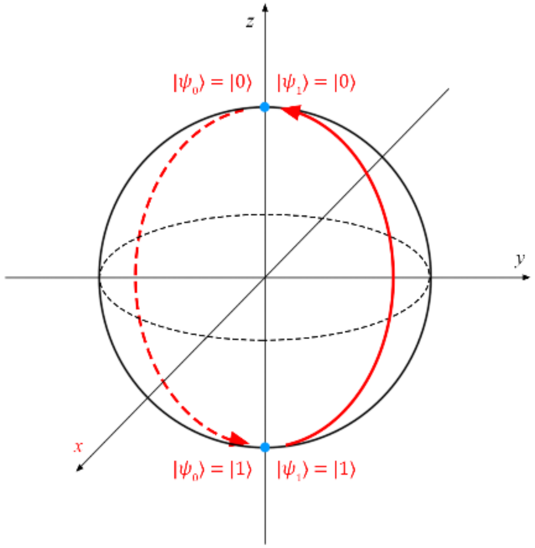


Pauli Y

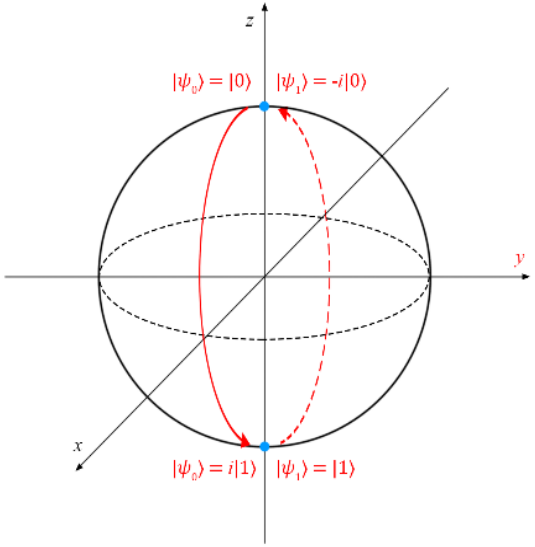
Pauli gates

$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ $\qquad$ $Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$ $\qquad$ $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

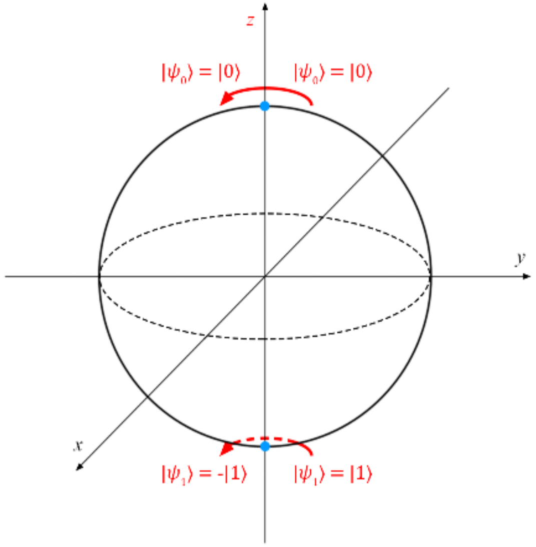
$X \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \beta \\ \alpha \end{pmatrix}$ $\qquad$ $Y \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = i \begin{pmatrix} -\beta \\ \alpha \end{pmatrix}$ $\qquad$ $Z \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha \\ -\beta \end{pmatrix}$



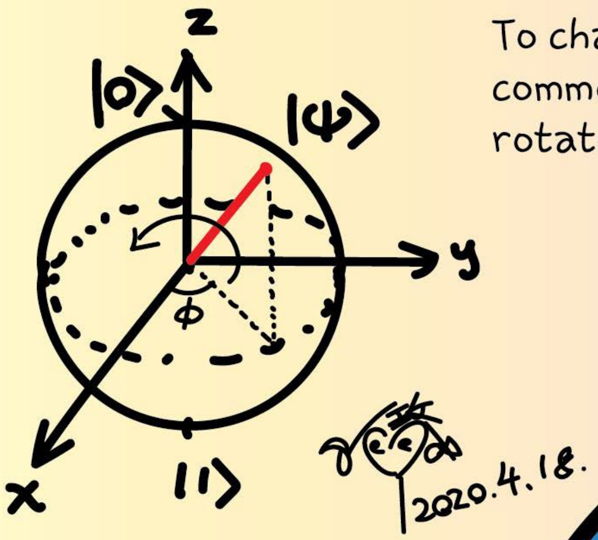
Pauli X



Pauli Y

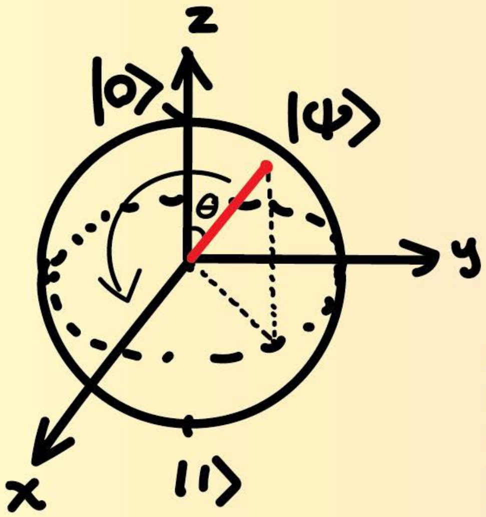


Pauli Z



To change the phase φ, we have a commonly used gate, Z, which rotates about the z-axis by 180°.

$$Z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



Similarly, the X gate rotates about the x-axis by 180°, rotating the angle θ e.g. $X|0\rangle = |1\rangle$, $X|1\rangle = |0\rangle$.

$$X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

We have seen in page 18 the two matrices for changing φ and θ in arbitraty amounts. They form a universal gate set – they can put a state anywhere on the Bloch Sphere. The gates Z and X are special cases of them.

Hadamard H

$$H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

Hadamard H

$$H = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}$$

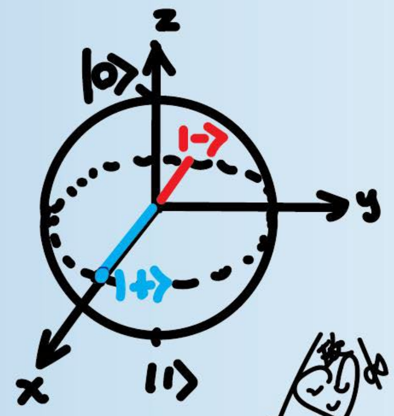
$$H|0\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} (|0\rangle + |1\rangle) \equiv |+\rangle$$

$$H|1\rangle = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$= \frac{1}{\sqrt{2}} (|0\rangle - |1\rangle) \equiv |-\rangle \,.$$



Another important gate is the H (or Hadamard) gate. It changes states $|0\rangle$ and $|1\rangle$ and creates two new states in between them:

$$H|0\rangle = |+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$$
$$H|1\rangle = |-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$$

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

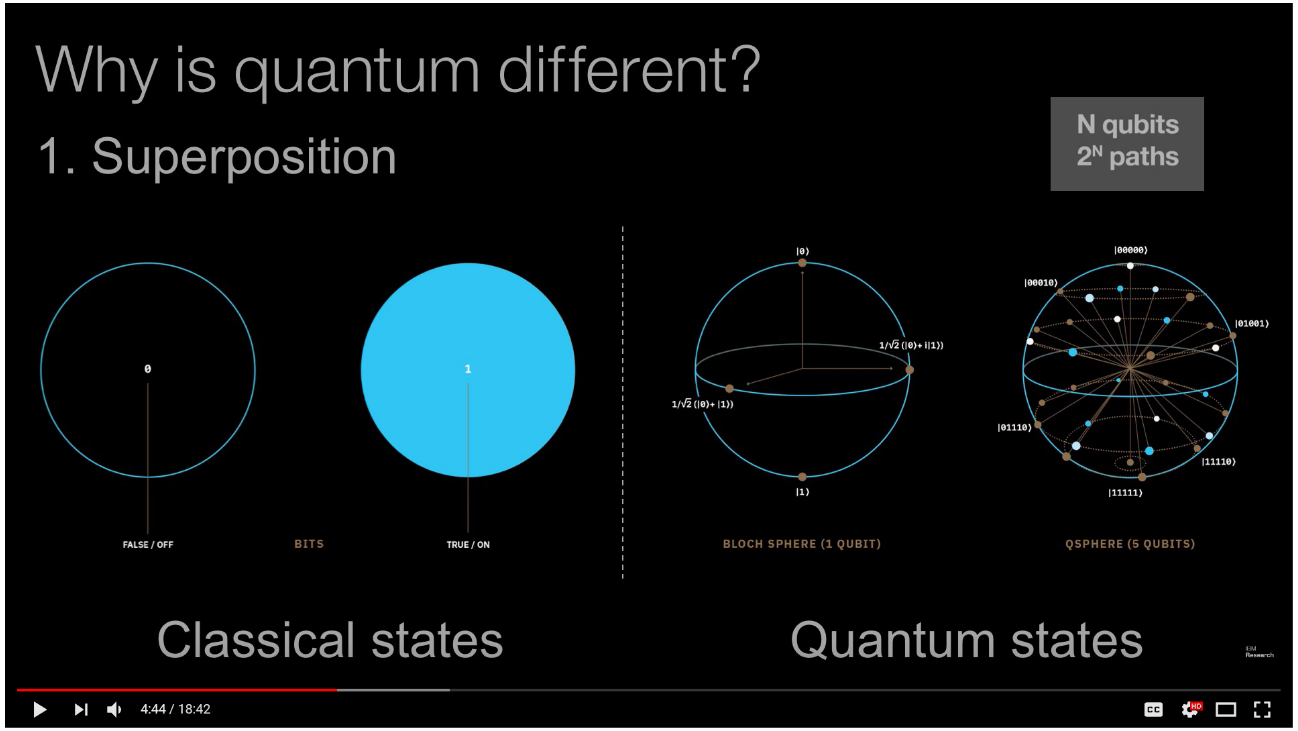
And some other commonly used gates:

$$S = \sqrt[2]{Z} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}$$ Rotates about z-axis by 90°

$$T = \sqrt[4]{Z} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix}$$ Rotates about z-axis by 45°

$$R8 = \sqrt[8]{Z} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/8} \end{pmatrix}$$ Rotates about z-axis by 22.5°

But these are all for a single qubit. What about gates for multiple qubits?



A Beginner's Guide to Quantum Computing

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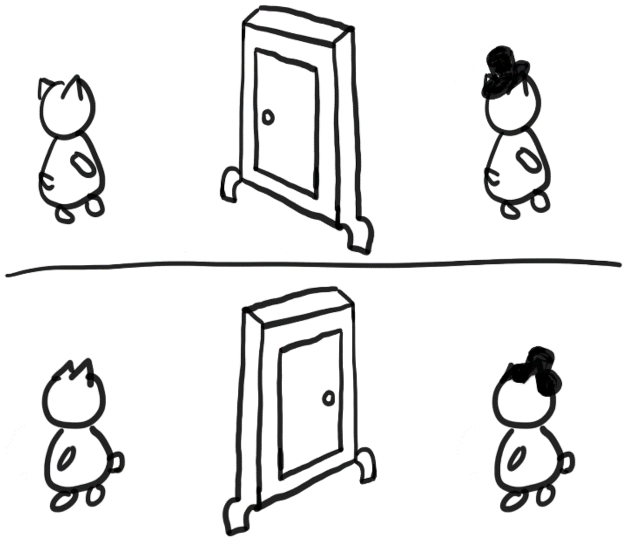
IBM Research
Published on May 31, 2017

Gates



manipulate qubit states (vectors)
through matrix multiplications

Unitarity $U^\dagger U = I$

So that it is reversible and probabilities add up to 1

Math insert – unitary, adjoint or Hermitian conjugate --

In math, unitarity means $U^\dagger U = I$, where I is the identity matrix and the "†" symbol (reads "dagger") means adjoint or Hermitian conjugate of matrix U. It can be further written as $U^\dagger = (U^*)^T = (U^T)^*$, where "*T*" denotes transpose and "*" complex conjugate:

$$\begin{pmatrix} U_1 \\ U_2 \\ \vdots \\ U_N \end{pmatrix}^T = (U_1 \quad U_2 \quad \dots \quad U_N)$$

and if $a = a_0 + ia_1$, then $a^* = a_0 - ia_1$ by definition. Therefore,

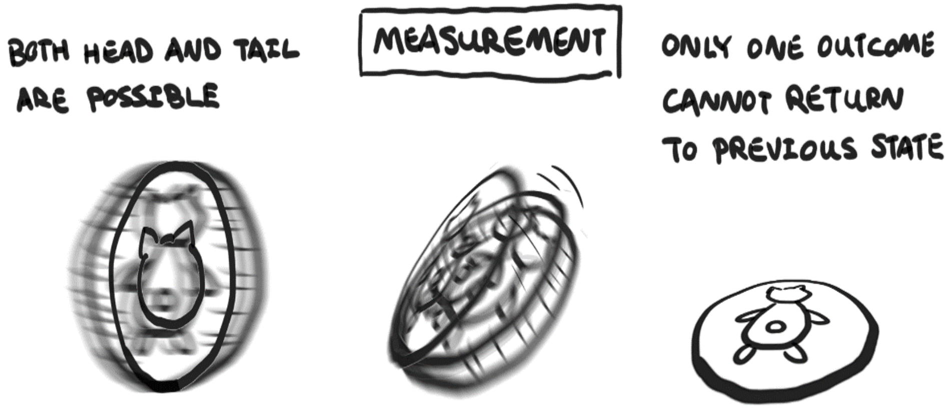
$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}^\dagger = \begin{pmatrix} a^* & c^* \\ b^* & d^* \end{pmatrix}.$$

Q# exercise: option 1

No installation, web-based Jupyter Notebooks

- The Quantum Katas project (tutorials and exercises for learning quantum computing) https://github.com/Microsoft/QuantumKatas

Measurement – not a gate



Not reversible

$$|\psi\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$

$$P = |c_{00}|^2 + |c_{01}|^2$$

If first qubit is 0

$$|\psi'\rangle = \frac{c_{00}|00\rangle + c_{01}|01\rangle}{\sqrt{P}}$$

After measurement

Measurement

If we use the wavefunction approach, we can derive the value we'd expect to measure for a large number of measurements of a given observable, M. The expectation value can be obtained as

$$\langle M \rangle = \langle \psi | M | \psi \rangle = \sum_j m_j \, |c_j|^2 \ ,$$

where m_j is each measurement result of M, and $|c_j|^2 = P(m_j)$ is the probability of getting result m_j. Obtaining m_j leaves the system in the state $|\psi_j\rangle$. This unavoidable disturbance of the system caused by the measurement process is often described as a "collapse," a "projection" or a "reduction" of the wavefunction.